Hypothesis testing: comparing proportions

- Prof. Giuseppe Verlato
- Unit of Epidemiology & Medical Statistics, Dept. of Diagnostics & Public Health, University of Verona

Mendel's experiment:

Mendel bred together smooth yellow peas (dominant traits) and wrinkled green peas (recessive traits), and further inbred the 1° generation of hybrids.

	Yellow	green	
Smooth	315	108	423
Wrinkled	101	32	133
	416	140	556

How many peas are **EXPECTED** in each cell under the hypothesis of statistical independence?

Expected in the 1st cell = p(smooth $\cap$ yellow) * N =

= p(smooth)*p(yellow)*N = (423/556)*(416/556)*556 =

= 423 * 416 / 556 = 316.5

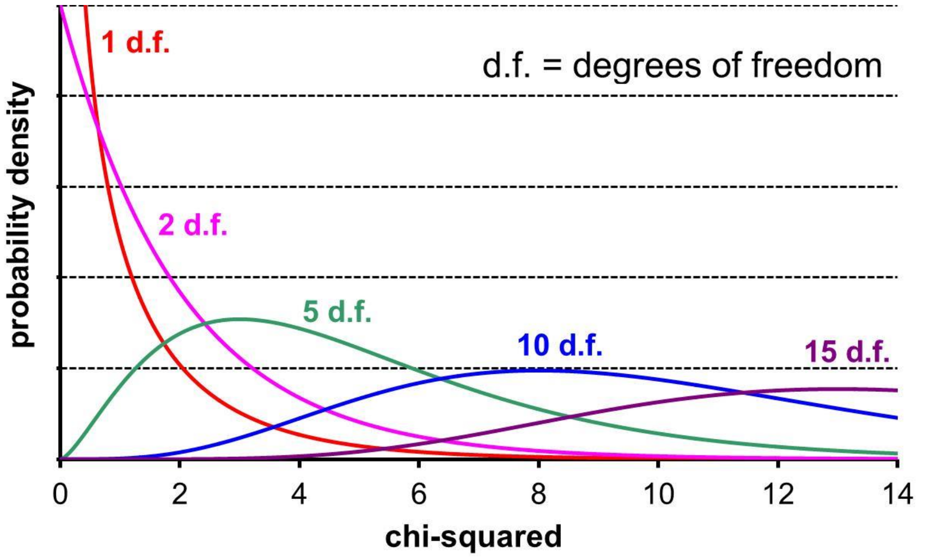
$$\text{Number of Expected} = \frac{\text{(row units)*(column units)}}{\text{overall units}}$$

OBSERVED	yellow	green	
smooth	315	108	423
wrinkled	101	32	133
	416	140	556

EXPECTED	yellow	green	
smooth	316.5	106.5	423
wrinkled	99.5	33.5	133
	416	140	556

At first sight the hypothesis of statistical independence between surface characteristic (smooth / wrinkled) and color (yellow/green) seems to be valid: genetic traits are independently assorted.

The chi-square (χ^2) test, based on the chi-square distribution, allows to verify the hypothesis of statistical independence **IN A SCIENTIFIC WAY**.



Chi-squared test for independence

$$\chi^2 = \Sigma \text{ (observed – expected)}^2 \text{ / expected}$$

H_0: the two variables are statistically independent
H_1: the two variables are statistically dependent

Significance level = 5%
Degrees of freedom = (n° rows - 1) * (n° columns - 1) =
= (2-1)*(2-1) = 1*1 = 1
Critical threshold = $\chi^2_{1,\ 0.05}$ = 3.84

$$\chi^2 = \frac{(315-316.5)^2}{316.5} + \frac{(108-106.5)^2}{106.5} + \frac{(101-99.5)^2}{99.5} + \frac{(32-33.5)^2}{33.5}$$

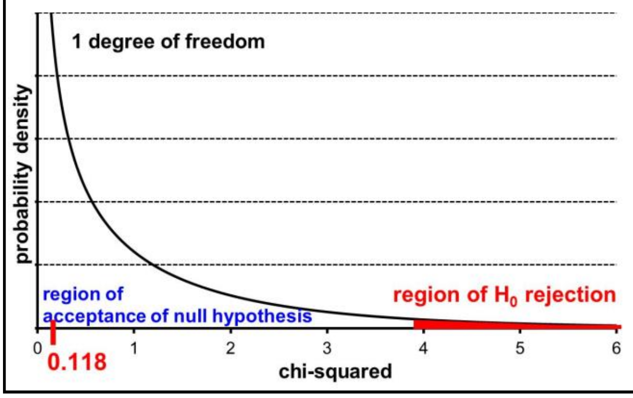
$$= 0.007 + 0.021 + 0.023 + 0.067 = 0.118$$

observed χ^2 < critical threshold

0.118 3.84

H_0 is accepted.

Traits "surface characteristic" and "color" are independently assorted (**3rd Mendel's law**).



N.B. chi-squared test is always one-sided.

Computing DEGREES OF FREEDOM of a CHI-SQUARE test

?	?	423
?	111	133
140	416	556

416-111

133-111

?	305	423
22	111	133
140	416	556

140-22

423-305

118	305	423
22	111	133
140	416	556

In a 2*2 table
DEGREES OF FREEDOM = 1

Computing DEGREES OF FREEDOM of a CHI-SQUARE test

?	?	?	463
?	72	56	193
100	140	416	656

	100-65	140-72	416-56	
463-68-360	35	68	360	463
193-72-56	65	72	56	193
	100	140	416	656

In a 2*3 table
DEGREES OF FREEDOM = 2

Computing DEGREES OF FREEDOM of a CHI-SQUARE test

?	?	423
?	111	133
140	416	556

degrees of freedom = 1

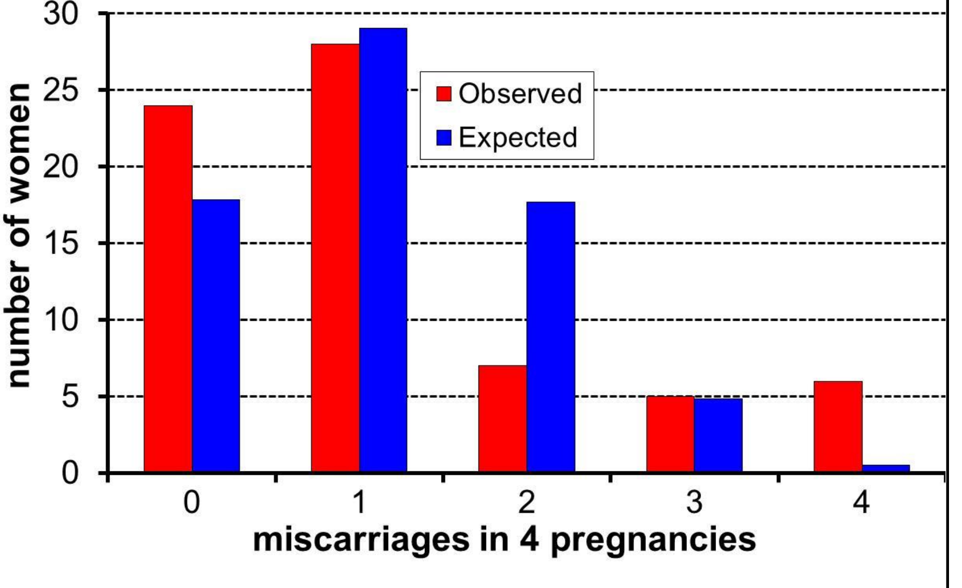
?	?	?	463
?	72	56	193
100	140	416	656

degrees of freedom = 2

Is there unifying formula ? YES
DEGREES OF FREEDOM =
(n ROWS – 1) * (n COLUMNS -1)

- The chi-square test is based on the normal approximation.
- The chi-square test is suited when there are at least 5 expected per cell in a 2*2 table.
- Otherwise, the Fisher's exact test must be used instead.

Chi-square test for goodness of fit



Chi-square test for goodness-of-fit

$$\chi^2 = \Sigma \text{ (observed – expected)}^2 \text{ / expected}$$

H_0: observed data follow a binomial distribution
H_1: observed data DO NOT follow a binomial distribution

Level of significance = 5%
Degrees of freedom =n° cells - n° parameters = 5-2 = 3

Critical threshold = $\chi^2_{3,\,0.05}$ = 7.81

$$\chi^2 = \frac{(24-17.9)^2}{17.9} + \frac{(28-29.1)^2}{29.1} + \frac{(7-17.7)^2}{17.7} + \frac{(5-4.8)^2}{4.8} + \frac{(6-0.5)^2}{0.5}$$

$$= 2.12 + 0.04 + 6.48 + 0.01 + 61.96 = 70.60$$

Observed χ^2 > critical threshold → H_0 is rejected
70.60 7.81

PAIRED DATA

It is sometimes necessary to measure the same variable several times. For instance, presence/ absence of allergic rhinitis can be assessed during spring or in winter.

McNemar's test = to test if there are significant differences on a dichotomous dependent variable between two related groups (2 measurements)

Cochran's Q test = to test if there are significant differences on a dichotomous dependent variable between two or more related groups (2 or more measurements)